Mr. Wright's Math Extravaganza

**Precalculus**
**Functions and Graphs**

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: 80% on test and success on applications

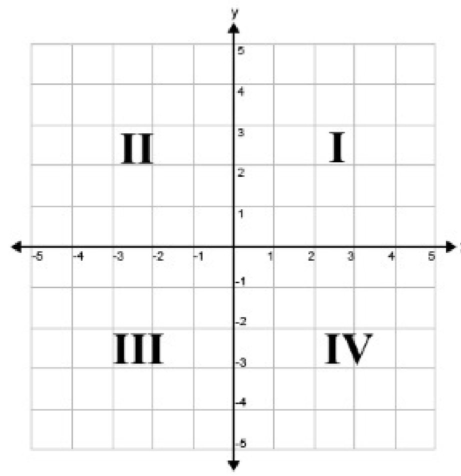
| Score | I Can Statements |
|---|---|
| 4.0 | ☐ I can demonstrate in-depth inferences and applications that go beyond what was taught. |
| *3.5* | In addition to score 3.0 performance, partial success at score 4.0 content. |
| 3.0 | ☐ I can find and interpret rate-of-change of functions.<br>☐ I can graph functions. |
| *2.5* | No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content. |
| 2.0 | ☐ I can use the Cartesian coordinate system. (Graphing, distance, and midpoint)<br>☐ I can graph equations and find *x*- and *y*- intercepts.<br>☐ I can write and graph linear functions.<br>☐ I can find the domain and range of a function.<br>☐ I can evaluate and graph piecewise functions.<br>☐ I can find the zeros and rate of change of a graph.<br>☐ I can identify the graphs of parent functions.<br>☐ I can graph using transformations.<br>☐ I can combine functions (+, −, ×, ÷, composition)<br>☐ I can find the inverse of a function.<br>☐ Find the best-fitting line for linear data. |
| *1.5* | Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content. |
| 1.0 | With help, partial success at score 2.0 content and score 3.0 content. |
| *0.5* | With help, partial success at score 2.0 content but not at score 3.0 content. |
| 0.0 | Even with help, no success. |

# Precalculus

## 1-01 The Cartesian Plane

### Cartesian Plane

- ___________________quadrants
- Point is ___________________




Graph A(3, 2)
Graph B(−1, 4)

### Distance formula

- ___________________Theorem
- $a^2 + b^2 = c^2$
- $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Midpoint formula

- ___________________of the points (mean)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the (a) distance and (b) midpoint between (−1, 3) and (2, −5)

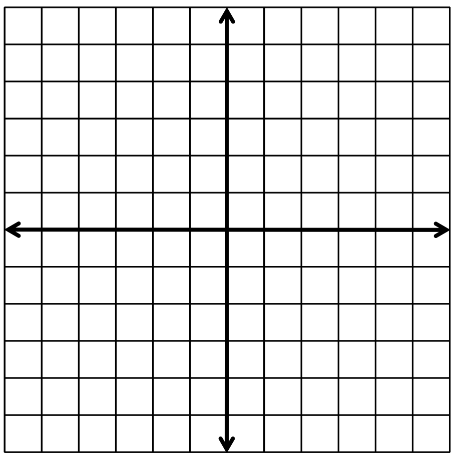
# Precalculus

## 1-02 Graphs

### Basic graphing method

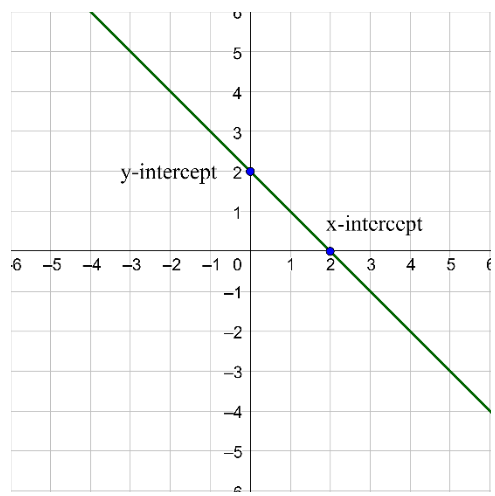
- Make a _______________
- Choose _______________, Calculate _______________

Graph $y = 3 - 0.5x$



### Intercepts

- Point where a graph _______________the axes
- To find the intercepts
  - $x$-intercept
    - Let _______________ and solve for $x$
  - $y$-intercept
    - Let _______________ and solve for $y$

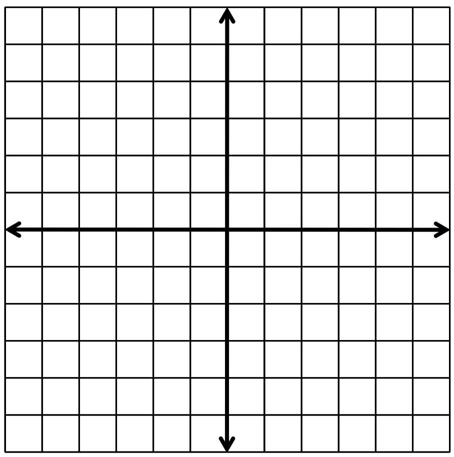



Find the intercepts of $y = 2x^2 + 2$

## Circles

- $(x-h)^2+(y-k)^2=r^2$
  - where __________________ is the center
  - __________________is the radius

Graph $(x+2)^2+(y-1)^2=4$



# Precalculus

1-03 Linear Equations in Two Variables

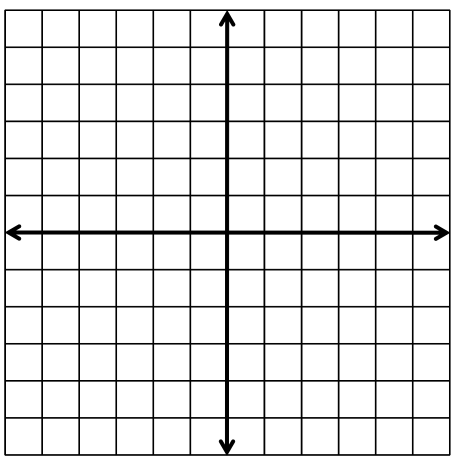
## Slope-intercept form

- $y = mx + b$
  - $m$ = ________________(rate of change)
  - $(0, b)$ = ________________
- $y = b \rightarrow$ ________________line
- $x = a \rightarrow$ ________________line

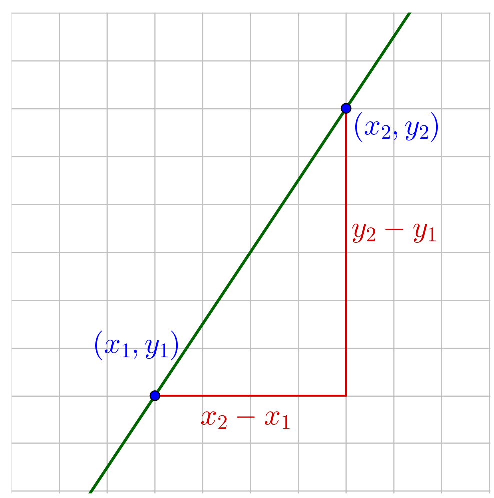
## To graph a line (shortcut)

1. Plot ________________
2. Follow the ________________to get a couple more points
3. Draw a ________________through the points

Find the slope and $y$-int and graph $y = 3x - 4$



## Slope




- $slope = \frac{rise}{run}$
- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- If slope is
  - $m > 0 \rightarrow$ ________________
  - $m = 0 \rightarrow$ ________________
  - $m < 0 \rightarrow$ ________________
  - $m$ undefined $\rightarrow$ ________________

Find the slope of the line passing through $(-3, -2)$ and $(1, 6)$

## Write Linear Equations

1. Find ________________ $(m)$
2. Find a ________________ on the line $(x_1, y_1)$
3. Use ________________ form $y - y_1 = m(x - x_1)$

Find slope-intercept form of the line passing through (2, 4) with $m$ = 3.

## Parallel and Perpendicular

- Parallel → ________________ slope
- Perpendicular → slopes are ________________ ________________
  - $m_1 \cdot m_2 = -1$

Find the equation of the line passing through (2, 1) and perpendicular to $4x - 2y = 3$.

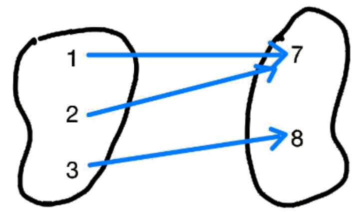
# Precalculus

1-04 Functions and Functional Notation

## Relation

- Rule that relates _______________

## Function




- Special _______________
- A function $f$ from set A to set B is a relation that assigns each element $x$ in set A to _______________one element in set B
- Set A: _______________, _______________
- Set B: _______________, _______________

Is this a function?

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-8$ | $-1$ | $0$ | $1$ | $8$ |

$x^2 + y = 4$

$x + y^2 = 16$

## Functional Notation

$f(x) = x^2 + 4$

_______________ _______________ _______________

If $f(y) = 3 - \sqrt{y}$, evaluate

$f(4)$

$f(4x^2)$

## Piecewise functions

- Function made of _______________function with specific _______________

$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

Evaluate $f(-1)$

$f(2)$

## Domain of a function

- Implied domain - all real numbers for which the expression is ________________

## Interval notation

- [ ] means ________________
- ( ) means ________________
- (2, 7] means ________________

What is the domain?

$h(t) = \frac{4}{t}$

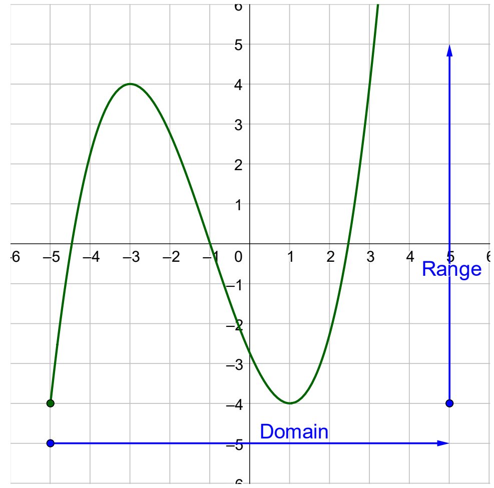
$f(x) = \sqrt{5x - 8}$

## Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Simplify the difference quotient for $f(x) = 2x + 1$

# Precalculus



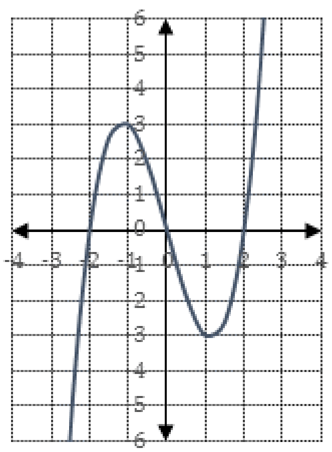

## 1-05 Graphs of Functions

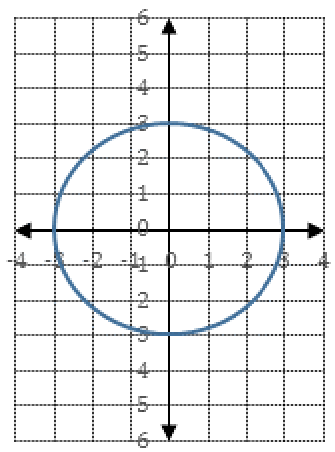
### Domain and range from a graph

- Domain: part of ____________________ covered by graph
- Range: part of ____________________ covered by graph

### Vertical Line Test

- A graph represents a function if ____________________ ____________________line can touch ____________________ on the graph





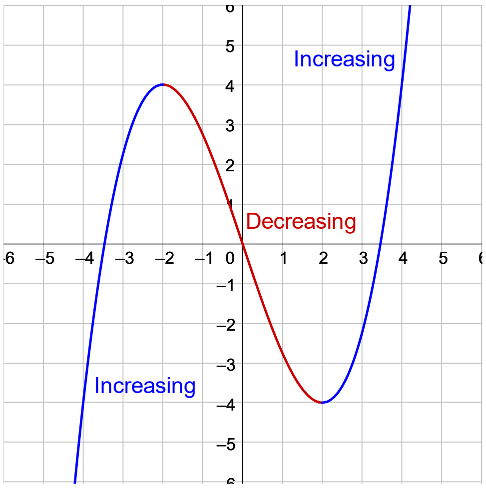
### Zeros of a function

- $x$-value such that ____________________
- ____________________
- To find, make $f(x) = 0$ and solve for $x$

Find the zeros of $f(x) = 2x^2 - 7x - 30$

## Analyzing Graphs

- Increasing (__________________from left to right)
- Decreasing (__________________from left to right)
- Constant (__________________)
- Relative minimum (__________________point in area)
- Relative maximum (__________________point in area)




## Rate of Change

- Average rate of change = __________________between 2 points

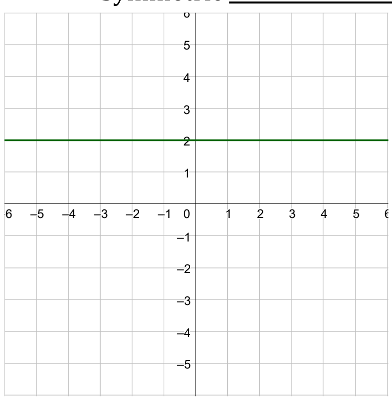
# Precalculus

## 1-06 Graphs of Parent Functions

### Parent Functions

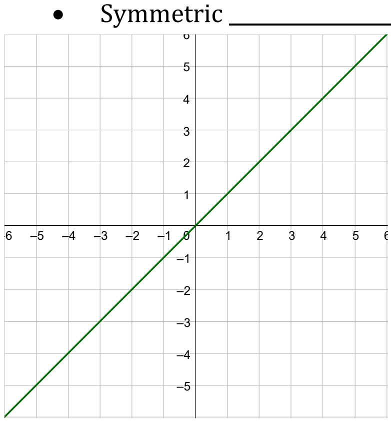
#### Constant Function $f(x) = c$,

- Domain is _______________.
- Range is _______________
- Neither increasing or decreasing.
- Symmetric _______________



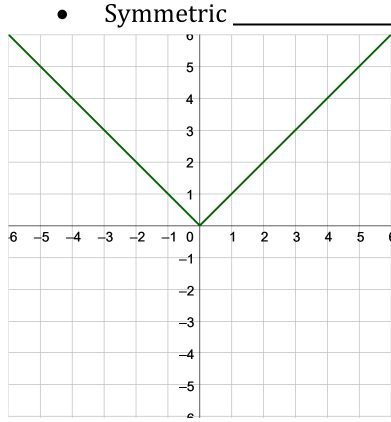
#### Linear Function $f(x) = x$,

- Domain is _______________
- Range is _______________
- Increases from $(-\infty, \infty)$.
- Symmetric _______________



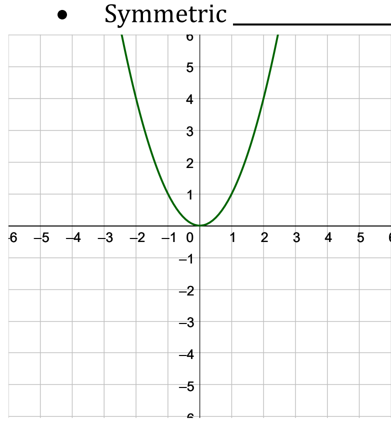
#### Absolute Value Function $f(x) = |x|$,

- Domain is _______________
- Range is _______________
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric _______________



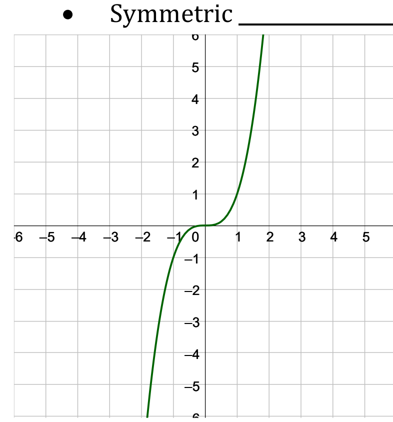
#### Quadratic Function $f(x) = x^2$,

- Domain is _______________
- Range is _______________
- Decreasing over $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric _______________



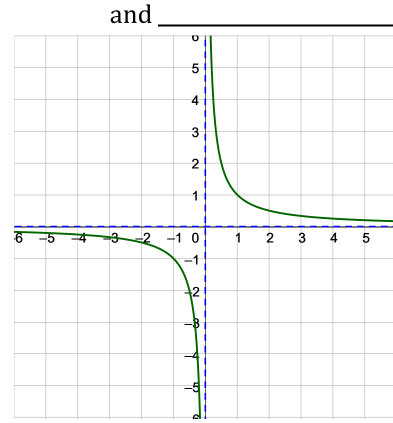
#### Cubic Function $f(x) = x^3$,

- Domain is _______________
- Range is _______________
- Increasing on $(-\infty, \infty)$.
- Symmetric _______________



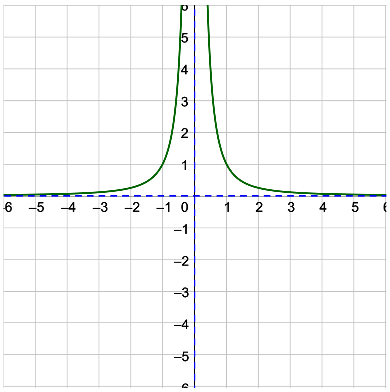
#### Reciprocal Function $f(x) = \frac{1}{x}$,

- Domain is _______________
- Range is _______________
- Decreasing on $(-\infty, 0)$ and $(0, \infty)$.
- Symmetric _______________ and _______________



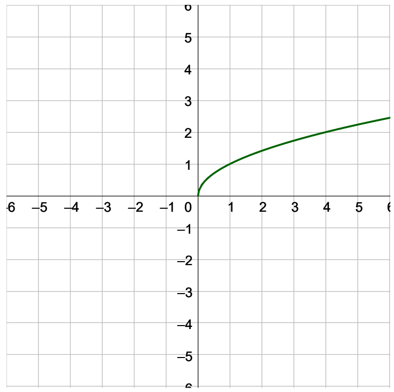
**Reciprocal Squared Function** $f(x) = \frac{1}{x^2}$,

- Domain is ________________
- Range is ________________
- Increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.
- Symmetric ________________



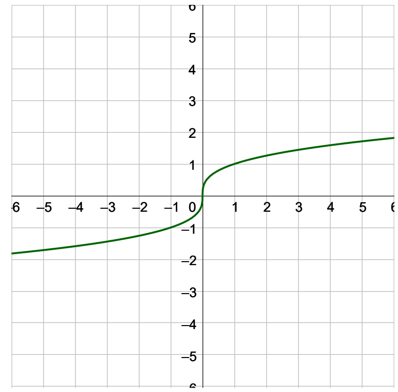
**Square Root Function** $f(x) = \sqrt{x}$,

- Domain is ________________
- Range is ________________.
- Increasing on $(0, \infty)$.
- Symmetric ________________



**Cube Root Function** $f(x) = \sqrt[3]{x}$,

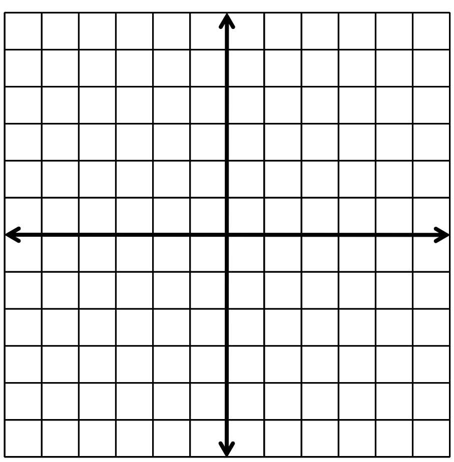
- Domain is ________________
- Range is ________________
- Increasing over $(-\infty, \infty)$.
- Symmetric ________________



## Piecewise Functions

- At the boundary,
  - If equal → ________________ dot
  - If not equal → ________________ dot

Graph $g(x) = \begin{cases} 3x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$



# Precalculus

## 1-07 Transformations of Functions

### Translations (shift)

- ________________the graph
- Horizontal
  - $g(x) = f(x - h)$
  - $h$ shifts ________________
- Vertical
  - $g(x) = f(x) + k$
  - $k$ shifts ________________

For $f(x) = |x|$, write a function with a vertical shift of 3 down and 2 right.

### Reflections

- Vertical
  - ________________
  - $g(x) = -f(x)$
- Horizontal
  - ________________
  - $g(x) = f(-x)$

### Dilations

- Stretch/Shrink
- Horizontal
  - $g(x) = f(bx)$
  - Stretch by ________________
- Vertical
  - $g(x) = af(x)$
  - Stretch by ________________

### Put it all together

$$g(x) = af(bx - h) + k$$

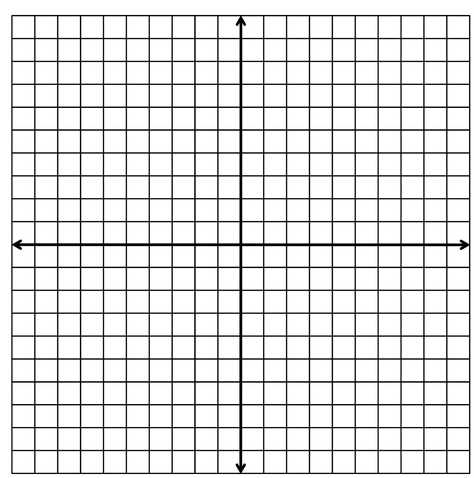
- $a$ = ________________stretch
- $\frac{1}{b}$ = ________________stretch
- $h$ = ________________shift right
- $k$ = ________________shift up

Given $g(x) = 2 - (x + 5)^2$

Identify the parent function

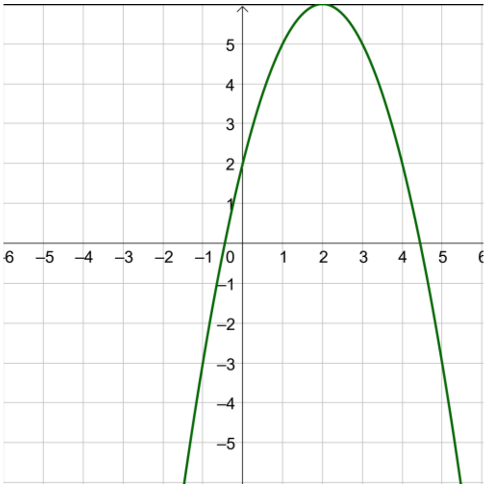
Describe the transformations

Sketch the graph



Use functional notation to write $g$ in terms of $f$

Write the function for



# Precalculus

1-08 Combinations of Functions

## Combining Functions

- Add $(f+g)(x)=f(x)+g(x)$
- Subtract $(f-g)(x)=f(x)-g(x)$
- Multiply $(fg)(x)=f(x)g(x)$
- Divide $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

If $f(x)=x+2$ and $g(x)=x-2$, find

$(f+g)(x)$

$(f-g)(x)$

$(fg)(x)$

$\left(\frac{f}{g}\right)(x)$

## Composition

- $(f\circ g)(x)=f\big(g(x)\big)$
- __________________ $g$ into $f$

If $f(x)=x^2$ and $g(x)=x-1$, find

$f\circ g$

$g\circ f$

- Domain of $(f\circ g)$ is all $x$ in domain of ______________ such that __________________ is in the domain of ______________.
- $x\to g\to f$

If $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{x}$, find the domain of $f\circ g$

## Decompose

- Find $f(x)$ and $g(x)$ so that $(f\circ g)(x)=h(x)$
- Pick a portion to be $g(x)$, then replace that with $x$ to get $f(x)$

Decompose $h(x)=2|x+3|$

Decompose $h(x)=\sqrt[3]{\frac{8-x}{5}}$

# Precalculus

## 1-09 Inverse Functions

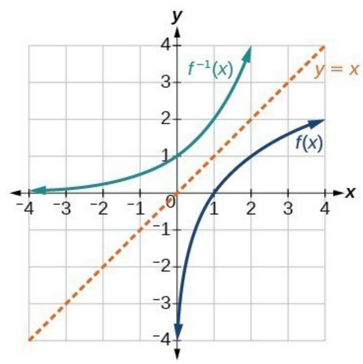
### Inverse functions

- Switch ________________
- Switch ________________and ________________
- Verify inverses by showing ________________and ________________

Verify that $f(x) = 7x - 4$ and $g(x) = \frac{x+4}{7}$ are inverses

### Graphs of inverses

- Reflected over line ________________




### One-to-one

- A function is one-to-one if each $y$ corresponds to ________________one $x$.
- Passes the ________________line test
- Inverse of a 1-to-1 is a ________________

### Finding inverses

1. ________________$f(x)$ with $y$
2. ________________$x$ and $y$
3. ________________for $y$
4. If you did step 1, ________________$y$ with $f^{-1}(x)$

Find the inverse of $f(x) = \sqrt[3]{10 + x}$

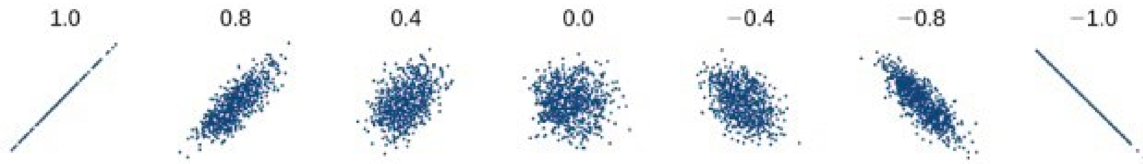
Find the inverse of $f(x) = x^2 - 2,\ x < 0$

# Precalculus

## 1-10 Mathematical Modeling

## Mathematical modeling

- Find a function to ___________________
- Least squares regression (___________________)
- Gives the ___________________line
- The amount of error is given by the ______________________________________($r$)




Number (in 1000s) of female USAF personnel, $P$, on active duty

| Year | 2000 | 2001 | 2002 | 2003 | 2004 |
|---|---|---|---|---|---|
| $P$ | 66.8 | 67.6 | 71.5 | 73.5 | 73.8 |

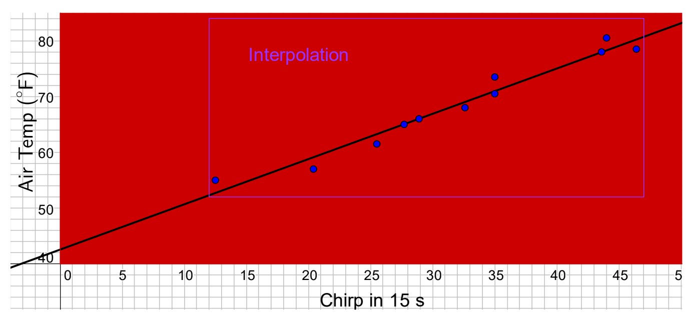
Find a model with $t = 0$ being 2000

## Real-Life Problems

- Slope = ___________________

## Interpolation and Extrapolation

- Interpolation
  - ___________________data
  - ___________________error
- Extrapolation
  - ___________________of data
  - ___________________error




## Variations

- Direct ___________________
  - $x\uparrow, y\uparrow$
- Inverse ___________________
  - $x\uparrow, y\downarrow$
- Joint ___________________
- $a$ = ___________________

A company found the demand for its product varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Write an equation.