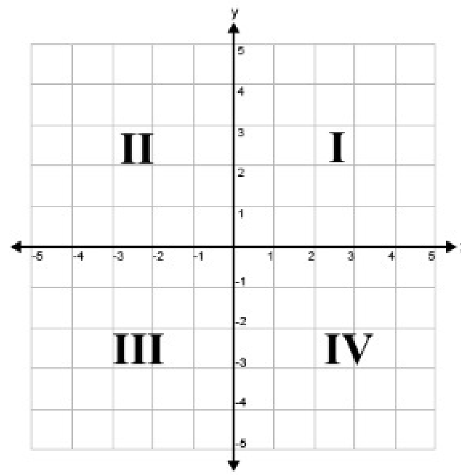
Precalculus

1-01 The Cartesian Plane

Cartesian Plane

- ___________________quadrants
- Point is ___________________

Graph A(3, 2)
Graph B(−1, 4)



Distance formula

- ___________________Theorem
- $a^2 + b^2 = c^2$
- $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

- ___________________of the points (mean)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the (a) distance and (b) midpoint between (−1, 3) and (2, −5)

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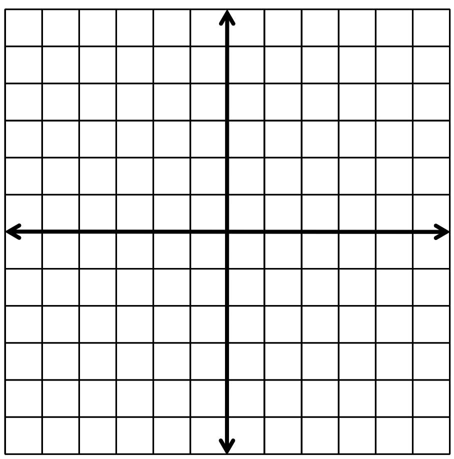
1-02 Graphs

Basic graphing method

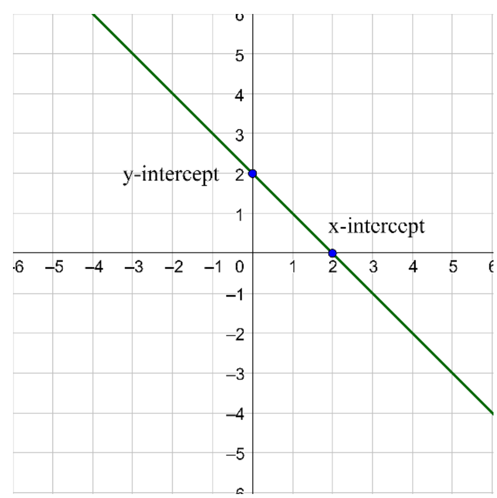
- Make a ________________
- Choose ________________, Calculate ________________

Graph $y = 3 - 0.5x$



Intercepts

- Point where a graph ________________the axes
- To find the intercepts
  - x-intercept
    - Let ________________ and solve for x
  - y-intercept
    - Let ________________ and solve for y

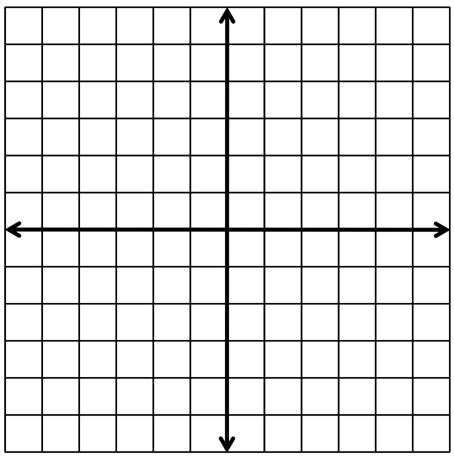


Find the intercepts of $y = 2x^2 + 2$

Circles

- $(x-h)^2+(y-k)^2=r^2$
  - where ___________________ is the center
  - ___________________is the radius

Graph $(x+2)^2+(y-1)^2=4$



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1-03 Linear Equations in Two Variables

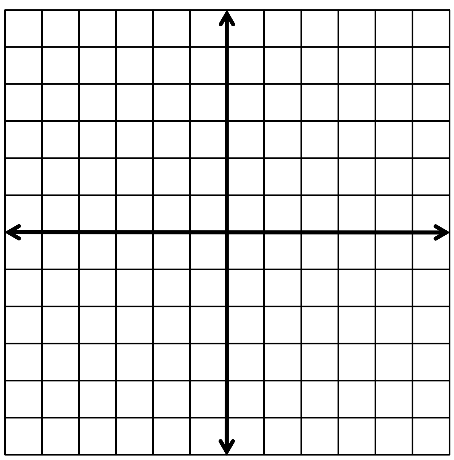
Slope-intercept form

- $y = mx + b$
  - m = ________________(rate of change)
  - $(0, b)$ = ________________
- $y = b$ → ________________line
- $x = a$ → ________________line

To graph a line (shortcut)

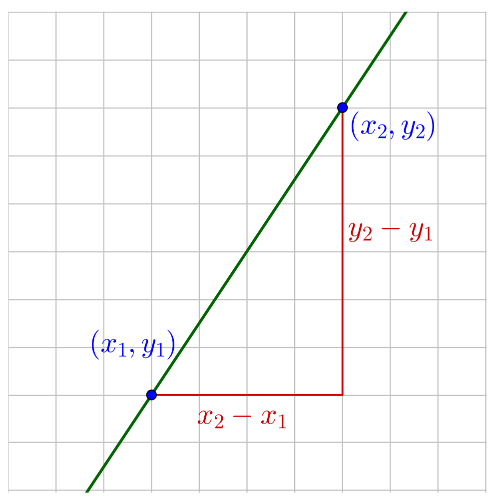
1. Plot ________________
2. Follow the ________________to get a couple more points
3. Draw a ________________through the points

Find the slope and y-int and graph $y = 3x - 4$



Slope

- $slope = \frac{rise}{run}$
- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- If slope is
  - $m > 0 \rightarrow$ ________________
  - $m = 0 \rightarrow$ ________________
  - $m < 0 \rightarrow$ ________________
  - m undefined → ________________



Find the slope of the line passing through (−3, −2) and (1, 6)

Write Linear Equations

1. Find ________________ (m)
2. Find a ________________ on the line (x_1, y_1)
3. Use ________________ form $y - y_1 = m(x - x_1)$

Find slope-intercept form of the line passing through (2, 4) with m = 3.

Parallel and Perpendicular

- Parallel → ________________ slope
- Perpendicular → slopes are ________________ ________________
  - $m_1 \cdot m_2 = -1$

Find the equation of the line passing through (2, 1) and perpendicular to $4x - 2y = 3$.

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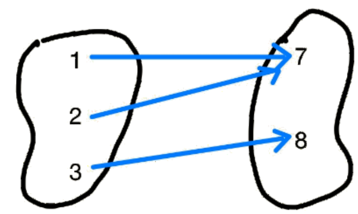
1-04 Functions and Functional Notation

Relation

- Rule that relates ________________

Function



- Special ________________
- A function f from set A to set B is a relation that assigns each element x in set A to ________________one element in set B
- Set A: ________________, ________________
- Set B: ________________, ________________

Is this a function?

x	-2	-1	0	1	2
y	-8	-1	0	1	8

$x^2 + y = 4$

$x + y^2 = 16$

Functional Notation

$$f(x) = x^2 + 4$$

________________ ________________ ________________

If $f(y) = 3 - \sqrt{y}$, evaluate

$f(4)$

$f(4x^2)$

Piecewise functions

- Function made of ________________function with specific ________________

$$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$$

Evaluate $f(-1)$

$f(2)$

Domain of a function

- Implied domain - all real numbers for which the expression is ________________

Interval notation

- [ ] means ________________
- ( ) means ________________
- (2, 7] means ________________

What is the domain?

$h(t) = \frac{4}{t}$

$f(x) = \sqrt{5x - 8}$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Simplify the difference quotient for $f(x) = 2x + 1$

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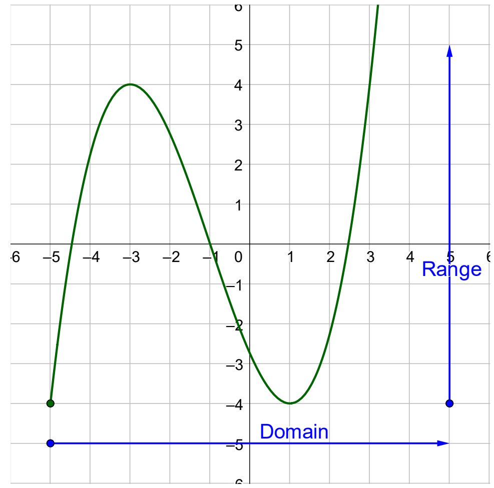
1-05 Graphs of Functions

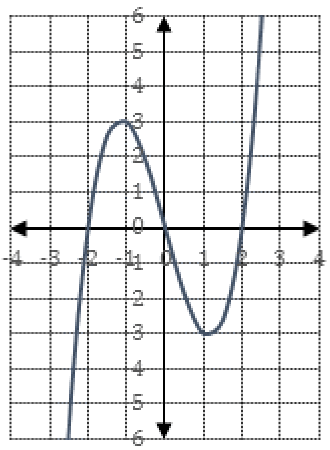


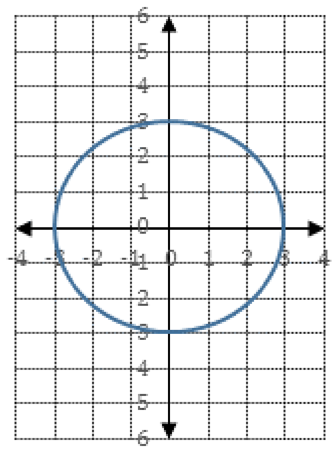
Domain and range from a graph

- Domain: part of ____________________ covered by graph
- Range: part of ____________________ covered by graph

Vertical Line Test

- A graph represents a function if ____________________ ____________________line can touch ____________________ on the graph





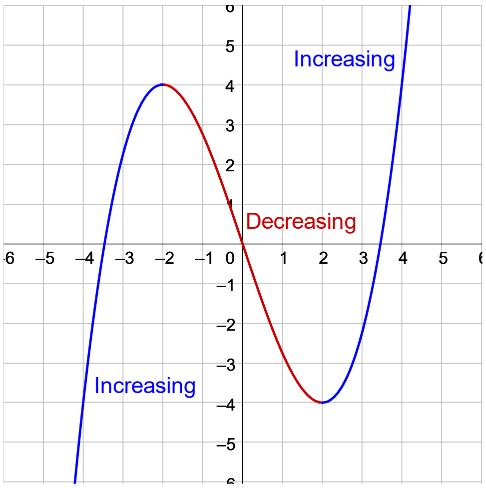
Zeros of a function

- x-value such that ____________________
- ____________________
- To find, make $f(x) = 0$ and solve for x

Find the zeros of $f(x) = 2x^2 - 7x - 30$

Analyzing Graphs

- Increasing (___________________from left to right)
- Decreasing (___________________from left to right)
- Constant (___________________)
- Relative minimum (___________________point in area)
- Relative maximum (___________________point in area)



Rate of Change

- Average rate of change = ___________________between 2 points

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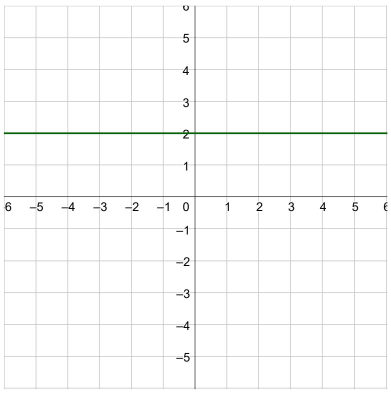
1-06 Graphs of Parent Functions

Parent Functions

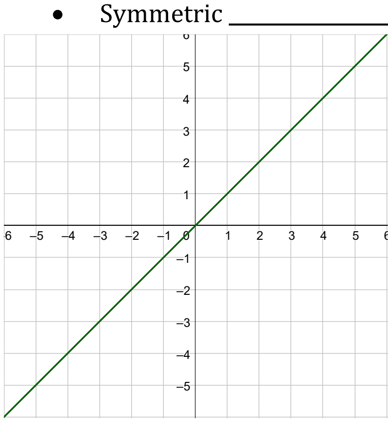
Constant Function $f(x) = c$,

- Domain is _______________.
- Range is _______________
- Neither increasing or decreasing.
- Symmetric _______________



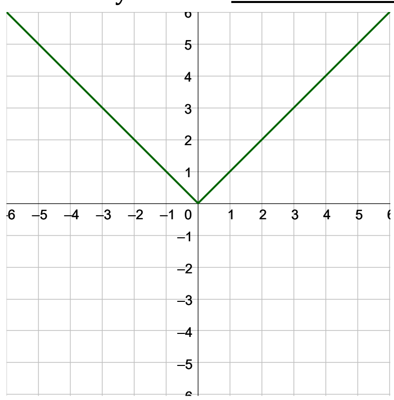
Linear Function $f(x) = x$,

- Domain is _______________
- Range is _______________
- Increases from $(-\infty, \infty)$.
- Symmetric _______________



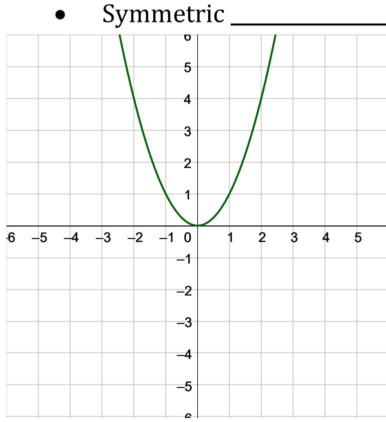
Absolute Value Function $f(x) = |x|$,

- Domain is _______________
- Range is _______________
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric _______________



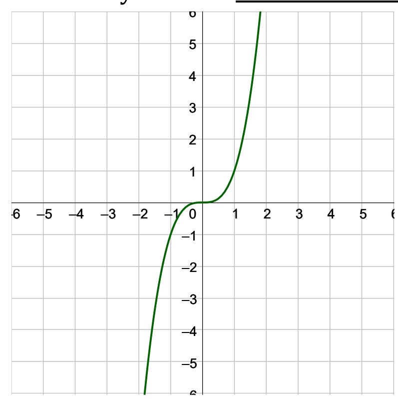
Quadratic Function $f(x) = x^2$,

- Domain is _______________
- Range is _______________
- Decreasing over $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric _______________



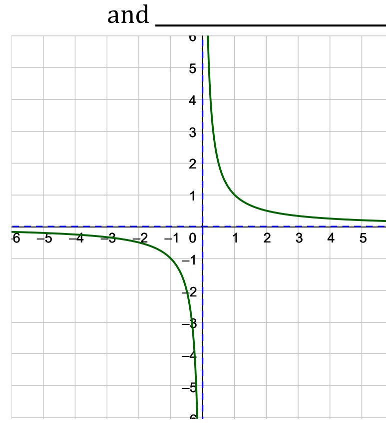
Cubic Function $f(x) = x^3$,

- Domain is _______________
- Range is _______________
- Increasing on $(-\infty, \infty)$.
- Symmetric _______________



Reciprocal Function $f(x) = \frac{1}{x}$,

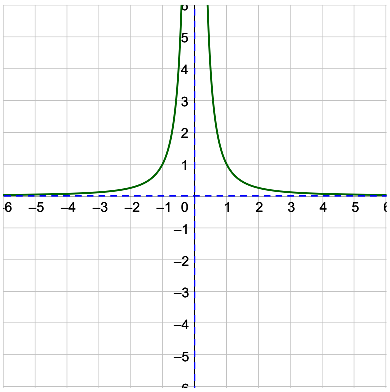
- Domain is _______________
- Range is _______________
- Decreasing on $(-\infty, 0)$ and $(0, \infty)$.
- Symmetric _______________ and _______________



Name: ________________

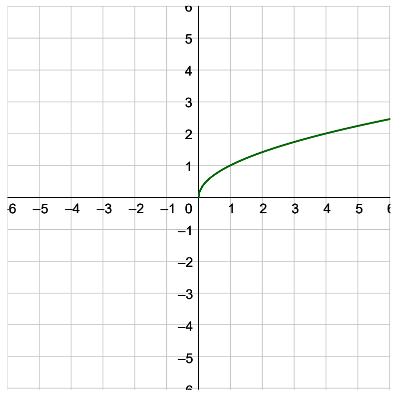
Reciprocal Squared Function $f(x) = \frac{1}{x^2}$,

- Domain is ________________
- Range is ________________
- Increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.
- Symmetric ________________



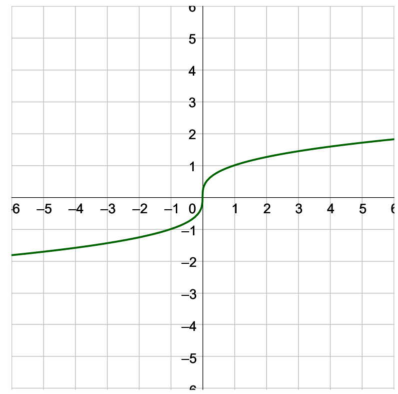
Square Root Function $f(x) = \sqrt{x}$,

- Domain is ________________
- Range is ________________.
- Increasing on $(0, \infty)$.
- Symmetric ________________



Cube Root Function $f(x) = \sqrt[3]{x}$,

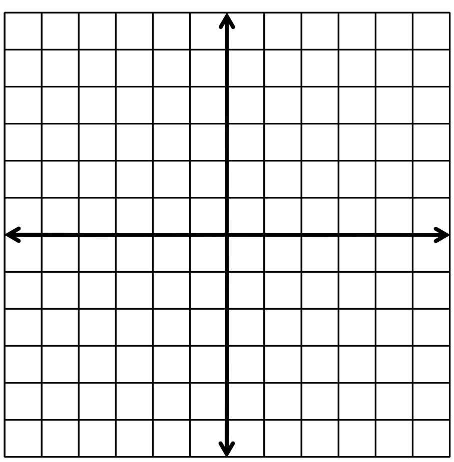
- Domain is ________________
- Range is ________________
- Increasing over $(-\infty, \infty)$.
- Symmetric ________________



Piecewise Functions

- At the boundary,
  - If equal → ________________ dot
  - If not equal → ________________ dot

Graph $g(x) = \begin{cases} 3x, & x \le 1 \\ x^2, & x > 1 \end{cases}$



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1-07 Transformations of Functions

Translations (shift)

- ________________the graph
- Horizontal
  - $g(x) = f(x - h)$
  - h shifts ________________
- Vertical
  - $g(x) = f(x) + k$
  - k shifts ________________

For $f(x) = |x|$, write a function with a vertical shift of 3 down and 2 right.

Reflections

- Vertical
  - ________________
  - $g(x) = -f(x)$
- Horizontal
  - ________________
  - $g(x) = f(-x)$

Dilations

- Stretch/Shrink
- Horizontal
  - $g(x) = f(bx)$
  - Stretch by ________________
- Vertical
  - $g(x) = af(x)$
  - Stretch by ________________

Put it all together

$$g(x) = af(bx - h) + k$$

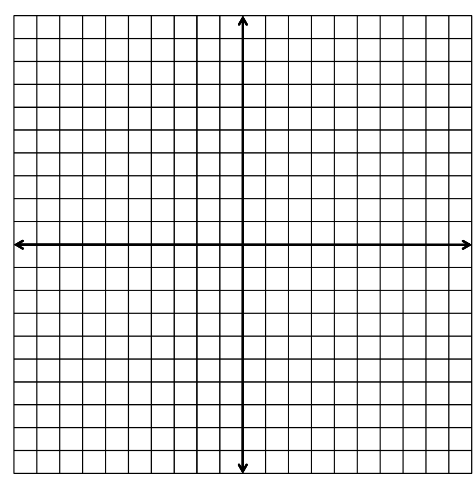
- a = ________________stretch
- $\frac{1}{b}$ = ________________stretch
- h = ________________shift right
- k = ________________shift up

Given $g(x) = 2 - (x + 5)^2$

Identify the parent function

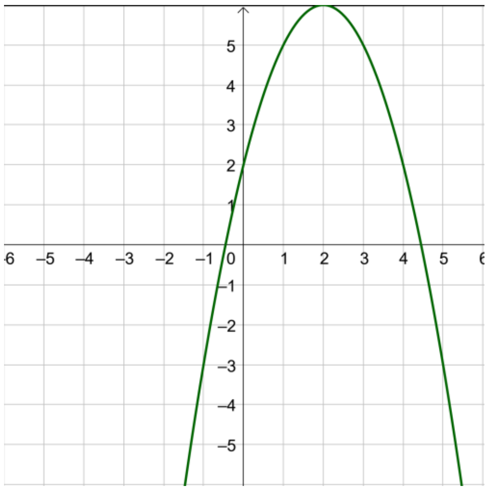
Describe the transformations

Sketch the graph



Use functional notation to write g in terms of f

Write the function for



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1-08 Combinations of Functions

Combining Functions

- Add $(f+g)(x) = f(x) + g(x)$
- Subtract $(f-g)(x) = f(x) - g(x)$
- Multiply $(fg)(x) = f(x)g(x)$
- Divide $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

If $f(x) = x + 2$ and $g(x) = x - 2$, find

$(f+g)(x)$

$(f-g)(x)$

$(fg)(x)$

$\left(\frac{f}{g}\right)(x)$

Composition

- $(f \circ g)(x) = f\big(g(x)\big)$
- ________________ g into f

If $f(x) = x^2$ and $g(x) = x - 1$, find

$f \circ g$

$g \circ f$

- Domain of $(f \circ g)$ is all x in domain of _____________ such that _______________ is in the domain of _____________.
- $x \to g \to f$

If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$, find the domain of $f \circ g$

Decompose

- Find $f(x)$ and $g(x)$ so that $(f \circ g)(x) = h(x)$
- Pick a portion to be $g(x)$, then replace that with x to get $f(x)$

Decompose $h(x) = 2|x + 3|$

Decompose $h(x) = \sqrt[3]{\frac{8-x}{5}}$

Precalculus

1-09 Inverse Functions

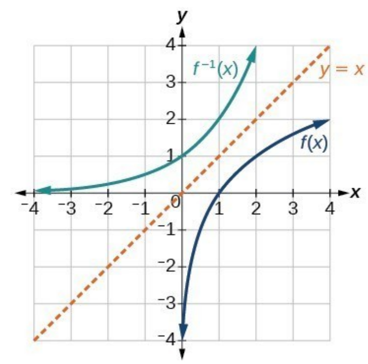
Inverse functions

- Switch _________________
- Switch _________________and _________________
- Verify inverses by showing _________________and _________________

Verify that $f(x) = 7x - 4$ and $g(x) = \frac{x+4}{7}$ are inverses

Graphs of inverses

- Reflected over line _________________



One-to-one

- A function is one-to-one if each y corresponds to _________________one x.
- Passes the _________________line test
- Inverse of a 1-to-1 is a _________________

Finding inverses

1. _________________$f(x)$ with y
2. _________________x and y
3. _________________for y
4. If you did step 1, _________________y with $f^{-1}(x)$

Find the inverse of $f(x) = \sqrt[3]{10 + x}$

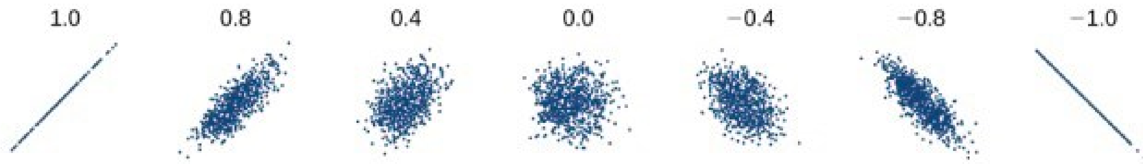
Find the inverse of $f(x) = x^2 - 2,\ x < 0$

Precalculus

1-10 Mathematical Modeling

Mathematical modeling

- Find a function to _______________
- Least squares regression (_______________)
- Gives the _______________line
- The amount of error is given by the _______________________________(r)



Number (in 1000s) of female USAF personnel, P, on active duty

Year	2000	2001	2002	2003	2004
P	66.8	67.6	71.5	73.5	73.8

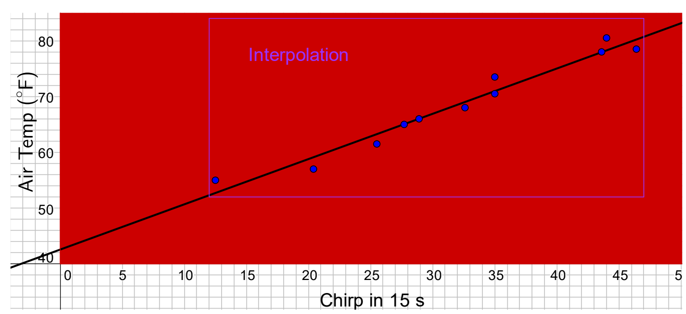
Find a model with $t = 0$ being 2000

Real-Life Problems

- Slope = _______________

Interpolation and Extrapolation

- Interpolation
  - _______________data
  - _______________error
- Extrapolation
  - _______________of data
  - _______________error



Variations

- Direct _______________
  - $x\uparrow, y\uparrow$
- Inverse _______________
  - $x\uparrow, y\downarrow$
- Joint _______________
- a = _______________

A company found the demand for its product varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Write an equation.